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AN INVENTORY MODEL WITH WEIBULL DISTRIBUTION DETERIORATION AND TIME DEPENDENT DEMAND

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ABSTRACT

In this paper, we developed a time dependent deteriorating inventory model with ramp type demand. Deterioration rate is taken as two parameters Weibull distribution. In this study, a more realistic scenario was assumed where part of the shortage was backordered and the rest was lost with time dependent backlogging rate. The environment of the whole study has been taken as inflationary. The whole combination of the setup is very unique and more practical. Finally, numerical example is presented to demonstrate the developed model and the solution procedure.

INTRODUCTION

In the literature of inventory after the development of classical economic order quantity (EOQ) model researchers extensively studied several aspects of inventory modeling by assuming constant demand rate. But in a real market demand of a product is always dynamic state due to the variability of time, price or even of the instantaneous level of inventory displayed in retail shop. This impressed researchers and marketing practitioners to think about the variability of demand rate. The ramp type demand is very commonly seen in real life situations when some fresh come to the market. In case of ramp type demand rate, the demand increases linearly at the beginning and then the market grows into a stable stage such that the demand becomes a constant until the end of the inventory cycle.

Most of the researches on real market oriented time dependent demand is very restrictive. Hill (1995) resolved the indiscipline of time dependent demand pattern by considering the demand as the combination of two different types of disciplined demand in two successive time periods over the entire time horizon and termed it as ramp – type time dependent demand pattern. The characteristic of ramp – type demand can be found in Mandal and Pal (1998). Order level inventory system with ramp-type demand rate has been taken for deterioration items. Wu et al. (1999) developed an EOQ model with ramp type demand rate for items with Weibull deterioration. Many researchers have been done on this subject. Wu and Ouyang (2000) extended the inventory model to include two different replenishment policies: (a) model started with no shortages and (b) modelstarting with shortage. Wu (2001) further investigated the inventory model with ramp type demand rate such that the deterioration followed the Weibull distribution. However, he did not guarantee the existence and uniqueness of the solution. Giri et al. (2003) noted a demand pattern for fashionable products which is increases exponentially with time for the seasonal products the steady rather than increasing exponentially. But for fashionable products as well as for the seasonal products the steady demand after its exponential increment with time never is continued indefinitely. Rather it would be followed by exponential decrement with respect to time after a period of time and becomes asymptotic in nature. Thus the demand would be

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illustrated by three successive time period classified time dependent ramp type function, in which demand increases with time and after that it becomes steady and towards the end in the final phase it decreases and becomes asymptotic. **Manna** and **Chaudhuri (2006)** have developed a production inventory model with ramp-type two time periods classified demand pattern where the finite production rate depends on the demand. **Singh et al. (2007)** developed an *EOQ* inventory model with Weibull distribution deterioration, ramp type demand and partial backlogging. **Deng et al. (2007)** point out some questionable results of **Mandal** and **Pal (1998)** and **Wu** and **Ouyang (2000)**, and then resolved the similar problem by offering a rigorous ad efficient method to derive the optimal solution. **Panda et al.** (2007) developed an EOQ inventory model with generalized ramp type demand and weibull distribution deterioration.

The perishable inventory model has been developed with a new pattern variable demand rate which is quadratic initially and becomes linear later on. In many real-life situations, the deterioration of goods is a realistic phenomenon. Certain products such as food stuff, blood bank, pharmaceuticals, volatile liquids and many others decrease continuously under deterioration due to vaporization, damage, spoilage, dryness etc. during their normal storage period. While determining the optimal inventory policy of that type of products, the loss due to deterioration can not be ignored. In the existing literature of inventory control the deteriorating inventory model has been developed, so as to accommodate more practical features of the real inventory systems. Therefore, it is realistic to consider the two parameter Weibull distribution deterioration rate.

In the most of the above referred models, complete backlogging of unsatisfied demand is assumed. In reality, often some customers are willing to wait and receive their orders at the end of shortage period, while others are more impatient and go elsewhere. In some inventory systems, such as fashionable commodities, the length of the waiting time for the next replenishment would determine whether the backlogging will be accepted or not. Therefore, the backlogging rate should be variable and depends on the waiting time for the next replenishment. To reflect this phenomenon, **Abad (2001)** discussed a pricing and lot–sizing problem for a product with a variable rate of deterioration, with shortages and partial backlogging. The backlogging rate depends on the time to replenishment. **Skouri** and **Papachristos** (2002) studied a multi period inventory model using the exponentially decreasing backlogging rate proposed by Abad. Recently many researchers have modified inventory policies by considering the "time proportional partial backlogging rate" such as **Wang** (2002), **Teng** and **Yang** (2004), **San Jose et al.** (2006) and **Wu et al.** (2006).

An inventory model for deteriorating items with ramp type demand under inflation has been developed. Two parameters Weibull distribution deterioration rate has been taken in this study. Shortages are allowed with partial backlogging. Backlogging rate is an exponential decreasing function of time. A numerical example is given to illustrate the results and the significant features of the results are discussed.

ASSUMPTIONS AND NOTATIONS

To develop the mathematical model of inventory with variable demand, partial backlogging and deterioration, the following notations and assumptions are used:

Notations:

1. q(t) is the inventory level at time t.

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- 2. A is the inventory ordering cost per order.
- 3. h is the holding cost per unit per unit time.
- 4. d is the deterioration cost per unit per unit time.
- 5. s is the shortage cost per unit per unit time.
- 6. ℓ is the unit cost of lost sales.
- 7. *S* is the initial inventory level after fulfilling backorders.
- 8. *Q* is the total amount of inventory ordered at the beginning of each replenishment cycle.
- 9. T is the length of replenishment cycle.
- 10. t_1 is the time at which shortage starts, $0 \le t_1 \le T$.

Assumptions:

- 1. Replenishment rate is infinite, and lead time is zero.
- 2. A single item is considered over the prescribed period of time.
- 3. There is no repair or replenishment of deteriorated units during the period.
- 4. Demand rate d(t) is assumed to be a function of time such that

 $d(t) = a + bt + c\{t - (t - \mu)H(t - \mu)\}t$, where $H(t - \mu)$ is the Heaviside's function defined as

follows $H(t - \mu) = \begin{cases} 1, & \text{if } t \ge \mu \\ 0, & \text{if } t < \mu \end{cases}$ and *a* is the initial rate of demand, **b** is the rate with which

the demand rate increases. The rate of change in demand rate itself increases at a rate c. a, b and c are positive constants.

- 5. $\theta(t) = \alpha \beta t^{\beta-1}$ be the variable rate of defective units out of on hand inventory at any time, where α is scale parameter and β is shape parameter and $0 < \alpha <<1$.
- 6. Shortages are allowed and unsatisfied demand is backlogged at a rate $e^{-i\delta}$, where the backlogging parameter δ is a positive constant.

FORMULATION AND SOLUTION OF THE MODEL

Using above assumptions, the inventory level during the interval $(0, t_1)$ is due to joint effect of variable demand and deterioration of items and the demand is partially backlogged in the interval (t_1,T) follows the pattern shown in the figure.



Figure 1: Graphical representation of the inventory system

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The inventory system is governed by the following differential equations in the interval (0, T) are $q'(t) + \alpha \beta t^{\beta-1} q(t) = -a + bt + ct^2 ,$ $0 \le t \le \mu$... (1) $\mu \le t \le t_1 \qquad \dots (2)$ $q'(t) + \alpha \beta t^{\beta-1} q(t) = -\left[a + b + c \mu t\right]$ and $q'(t) = -a + mt e^{-\delta t}$, where $m = b + c\mu$ $t_1 \leq t \leq T$... (3) with the conditions q 0 = S and $q(t_1) = 0$... (4) Solutions of equations (1) and (2) by using (4) are given by $q(t) = -\left(at + b\frac{t^{2}}{2} + c\frac{t^{3}}{3} + a\alpha \frac{t^{\beta+1}}{\beta+1} + b\alpha \frac{t^{\beta+2}}{\beta+2} + c\alpha \frac{t^{\beta+3}}{\beta+3}\right)e^{-\alpha t^{\beta}} + Se^{-\alpha t^{\beta}}$ $q(t) = \left(at_1 + \frac{t_1^2}{2} + a\alpha - \frac{t_1^{\beta+1}}{\beta+1} + m\alpha - \frac{t_1^{\beta+2}}{\beta+2} - at - m\frac{t^2}{2} - a\alpha - \frac{t^{\beta+1}}{\beta+1} - m\alpha - \frac{t^{\beta+2}}{\beta+2}\right)e^{-\alpha t^{\beta}},$ $\mu \le t \le t_1 \dots (6)$ From (5) and (6), we get $S = \left(at_1 + \frac{m_1^2}{2} + a\alpha - \frac{t_1^{\beta+1}}{\beta+1} + m\alpha - \frac{t_1^{\beta+2}}{\beta+2} - c\frac{\mu^3}{6} - c\alpha - \frac{\mu^{\beta+3}}{\beta+2}\right) \dots (7)$ Using (7), (5) becomes $q(t) = \left(at_1 + \frac{m_1^2}{2} + a\alpha - \frac{t_1^{\beta+1}}{\beta+1} + m\alpha - \frac{t_1^{\beta+2}}{\beta+2} - c \frac{\mu^3}{6} - c\alpha - \frac{\mu^{\beta+3}}{\beta+2} + 3 - c\alpha + \frac{\mu^{\beta+3}}{\beta+2} - c\alpha + \frac{\mu^{\beta+3}}{\beta+2} + 3 - c\alpha + \frac{\mu^{\beta+3}}{\beta+2} - \alpha + \frac{\mu^$ $-at - b\frac{t^{2}}{2} - c\frac{t^{3}}{3} - a\alpha \frac{t^{\beta+1}}{\beta+1} - b\alpha \frac{t^{\beta+2}}{\beta+2} - c\alpha \frac{t^{\beta+3}}{\beta+3} e^{-\alpha t^{\beta}},$ $0 \le t < \mu \dots (8)$ Also the solution of equation (3) by using (4) is given by $q(t) = \begin{bmatrix} a & (e_{-\delta t} - e^{-\delta t_1}) + a & (te_{-\delta t} - t_1 e^{-\delta t_1}) + a & (e_{-\delta t} - e^{-\delta t_1}) \\ a & \delta & \delta & (e_{-\delta t} - e^{-\delta t_1}) \end{bmatrix}, \qquad t_1 \le t \le T \qquad \dots (9)$ The holding cost during the period $(0,t_1)$ is given by $HC = h \left| \int_{0}^{\mu} q(t) e^{-rt} dt + \int_{0}^{t} q(t) e^{-rt} dt \right|$

$$=h\left[\frac{m\mu^{3}}{3}-\frac{c\mu^{4}}{4}-\frac{2c\alpha\mu^{\beta+4}}{(\beta+2)(\beta+4)}-\frac{b\mu^{3}}{6}+\frac{b\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{rc\mu^{5}}{12}\right]$$
$$+\frac{3c\alpha\mu^{\beta+5}}{2(\beta+2)(\beta+5)}+\frac{br\mu^{4}}{8}+\frac{c\mu^{5}}{15}+\frac{(b-m)\alpha\mu^{\beta+4}}{(\beta+2)(\beta+4)}-\frac{\alpha\mu\mu^{\beta+3}}{2(\beta+3)}+\frac{(\beta+2)\alpha\mu^{(\beta+4)}}{2(\beta+1)(\beta+4)}$$

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$$+\frac{at_{1}^{2}}{2}+\frac{mt_{1}^{3}}{3}-\frac{m\mu t_{1}^{2}}{2}\frac{2m\alpha\beta t_{1}^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{a\alpha\beta t_{1}^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{m\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)}$$
$$-\frac{art_{1}^{3}}{6}-\frac{mrt_{1}^{4}}{2}-\frac{ar\alpha t_{1}^{\beta+3}}{2(\beta+3)}-\frac{(\beta+6)m\alpha rt_{1}^{\beta+4}}{2(\beta+2)(\beta+4)}-\frac{mr\mu^{4}}{8}+\frac{m\alpha t_{1}^{2}\mu^{\beta+1}}{(\beta+1)}\right] \dots(10)$$

The deterioration cost during the period $(0,t_1)$ is given by

$$DC = d \left[\int_{0}^{\mu} \alpha \beta t^{\beta} e^{-n} q(t) dt + \int_{\mu}^{t} \alpha \beta t^{\beta} e^{-n} q(t) dt \right]$$

$$= d\alpha \beta \left[\frac{rc\mu^{\beta+4}}{6(\beta+1)} - \frac{b\mu^{\beta+2}}{2(\beta+2)} - \frac{c\mu^{\beta+3}(\beta+1)}{2\beta(\beta+3)} - \frac{rmt_{1}^{2}\mu^{\beta+1}}{2(\beta+1)} + \frac{br\mu^{\beta+3}}{2(\beta+3)} + \frac{c\mu^{\beta+4}}{3(\beta+4)} + \frac{at_{1}^{\beta+1}}{\beta(\beta+1)} + \frac{mt_{1}^{\beta+2}}{\beta(\beta+2)} + \frac{m\mu^{\beta+2}}{2(\beta+2)} - \frac{art_{1}^{\beta+2}}{(\beta+2)(\beta+1)} - \frac{mrt_{1}^{\beta+2}}{2(\beta+1)} + \frac{mrt_{1}\mu^{\beta+1}}{2(\beta+1)} + \frac{mrt_{1}^{\beta+3}}{2(\beta+1)} - \frac{mr\mu^{\beta+3}}{2(\beta+3)} \right] \dots (11)$$

The shortage cost during the period (t_1,T) is given by

$$SC = -s \left[\int_{t_{1}}^{T} q(t) e^{-rt} dt \right]$$

= $s \left[\frac{a}{\delta} \left(-\frac{e^{-\delta t_{1}} (e^{-rT} - e^{-rt_{1}})}{r} + \frac{(e^{-(\delta+r)T} - e^{-(\delta)t_{1}})}{(\delta+r)} \right) + \frac{m}{\delta} \left(-\frac{t_{1}e^{-\delta t_{1}} (e^{-rT} - e^{-rt_{1}})}{r} + \frac{(re^{-(\delta+r)T} - t_{1}e^{-(\delta+r)t_{1}})}{(\delta+r)^{2}} + \frac{(e^{-(\delta+r)T} - e^{-(\delta)t_{1}})}{(\delta+r)^{2}} + \frac{m}{\delta^{2}} \left(-\frac{e^{-\delta t_{1}} (e^{-rT} - e^{-rt_{1}})}{r} + \frac{(e^{-(\delta+r)T} - e^{-(\delta)t_{1}})}{(\delta+r)} \right) \right] \qquad \dots (12)$

The opportunity cost due to lost sales during the period (t_1,T) is given by

$$LSC = \ell \int_{t_{1}}^{t_{1}} a + mt (1 e^{-t}) \bar{e}^{\delta t} dt^{-}$$

$$= \ell \left[\frac{ae^{-rt_{1}}}{r} - \frac{ae^{-rT}}{r} + \frac{mt_{1}e^{-rt_{1}}}{r} - \frac{mTe^{-rT}}{r} + \frac{me^{-rt_{1}}}{r^{2}} - \frac{me^{-rT}}{r^{2}} - \frac{ae^{-(\delta+r)t_{1}}}{(\delta+r)} + \frac{ae^{-(\delta+r)T}}{(\delta+r)} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)} + \frac{mTe^{-(\delta+r)T}}{(\delta+r)^{2}} + \frac{me^{-(\delta+r)T}}{(\delta+r)^{2}} \right] \qquad \dots (13)$$

The ordering cost is given by

OC = A

... (14)

The total cost of the system is given by

The total average cost of the inventory system per unit time is given by

$$C_{A1}t,T = \frac{OC + HC + DC + SC + LSC}{T}$$

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$$= \frac{A}{T} + \frac{h}{T} \left[\frac{m \mu^{3}}{3} - \frac{c\mu^{4}}{4} - \frac{2c\alpha\mu^{\beta+4}}{(\beta+2)(\beta+4)} - \frac{b\mu^{3}}{6} + \frac{b\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{rc\mu^{5}}{12} \right] \\ + \frac{3c\alpha}{2(\beta+2)(\beta+5)} + \frac{br\mu^{4}}{8} + \frac{c\mu^{5}}{15} + \frac{(b-m)\alpha}{(\beta+2)(\beta+4)} - \frac{\alpha m\mu^{\beta+3}}{2(\beta+3)} + \frac{(\beta+2)\alpha}{2(\beta+3)} + \frac{\mu^{(\beta+4)}}{2(\beta+1)(\beta+4)} \\ + \frac{at_{1}^{2}}{2} + \frac{mt_{1}^{3}}{3} - \frac{m\mu t_{1}^{2}}{2} \frac{2m\alpha\beta t_{1}^{\beta+3}}{(\beta+1)(\beta+3)} + \frac{a\alpha\beta t_{1}^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{m\alpha\beta\mu^{\beta+3}}{2(\beta+2)(\beta+3)} + \frac{m\alpha\beta}{2(\beta+2)(\beta+3)} + \frac{m\alpha\beta}{$$

$$-\frac{art_{1}^{3}}{6} - \frac{mrt_{1}^{4}}{2} - \frac{ara_{1}^{\beta+3}}{2(\beta+3)} - \frac{(\beta+6)mart_{1}^{\beta+4}}{2(\beta+2)(\beta+4)} - \frac{mr\mu^{4}}{8} + \frac{mat_{1}^{2}\mu^{\beta+1}}{(\beta+1)}\right] \\ + \frac{d\alpha\beta}{T} \left[\frac{rc\mu^{\beta+4}}{6(\beta+1)} - \frac{b\mu^{\beta+2}}{2(\beta+2)} - \frac{c\mu^{\beta+3}(\beta+1)}{2\beta(\beta+3)} - \frac{rmt_{1}^{2}\mu^{\beta+1}}{2(\beta+1)} + \frac{br\mu^{\beta+3}}{2(\beta+3)} \right] \\ 3(-4) - \frac{1}{(-1)} - \frac{1}{(-2)} - \frac{1}{2(-2)} - \frac{c\mu^{\beta+3}(\beta+1)}{2\beta(\beta+3)} - \frac{rmt_{1}^{2}\mu^{\beta+1}}{2(\beta+1)} + \frac{br\mu^{\beta+3}}{2(\beta+3)} \right] \\ + \frac{mrt_{1}\mu^{\beta+1}}{2(\beta+1)} + \frac{mrt_{1}^{\beta+3}}{2(\beta+3)} - \frac{mr\mu^{\beta+3}}{2(\beta+3)} \right] + \frac{s}{T} \left[\frac{a}{\delta} \left(- \frac{e^{-\delta t_{1}}(e^{-\tau T} - e^{-\tau t_{1}})}{r} + \frac{(e^{-(\delta+r)T} - e^{-(\delta)t_{1}})}{(\delta+r)} \right) \right] \\ + \frac{m}{\delta} \left(- \frac{t_{1}e^{-\delta t_{1}}(e^{-\tau T} - e^{-\tau t_{1}})}{r} + \frac{(re^{-(\delta+r)T} - t_{1}e^{-(\delta+r)t_{1}})}{(\delta+r)} + \frac{(e^{-(\delta+r)T} - e^{-(\delta)t_{1}})}{(\delta+r)} \right) \right] \\ + \frac{m}{\delta^{2}} \left(- \frac{e^{-\delta t_{1}}(e^{-\tau T} - e^{-\tau t_{1}})}{r} + \frac{(e^{-(\delta+r)T} - e^{-(\delta)t_{1}})}{(\delta+r)} \right) \right] \\ + \frac{l}{T} \left[\frac{ae^{-\tau t_{1}}}{r} - \frac{ae^{-\tau T}}{r} + \frac{mt_{1}e^{-\tau t_{1}}}{r} - \frac{mTe^{-\tau T}}{r} + \frac{me^{-\tau t_{1}}}{r^{2}} - \frac{me^{-\tau T}}{r^{2}} - \frac{ae^{-(\delta+r)t_{1}}}{(\delta+r)} + \frac{ae^{-(\delta+r)T}}{(\delta+r)} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)} + \frac{mt_{1}e^{-(\delta+r)T}}{(\delta+r)} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)} + \frac{mt_{1}e^{-(\delta+r)T}}{(\delta+r)} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)} + \frac{mt_{1}e^{-(\delta+r)T}}{(\delta+r)} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)t_{1}}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)}}{(\delta+r)^{2}} - \frac{mt_{1}e^{-(\delta+r)}}{(\delta+r)$$

To minimize total average cost per unit time, the optimal values of t_1 and T can be obtained by solving the following equations simultaneously.

$$\frac{\partial C_A(t_1,T)}{\partial t_1} = 0 \qquad \dots (16)$$
$$\frac{\partial C_A(t_1,T)}{\partial T} = 0 \qquad \dots (17)$$

Provided, they satisfy the following conditions:

$$\frac{\partial^{2}C_{A}(t,T)}{\partial t_{1}^{2}} > 0, \frac{\partial^{2}C_{A}(t,T)}{\partial T^{2}} > 0$$

$$\left(\frac{\partial^{2}C(t,T)}{\partial t_{1}^{2}}\right) \left(\frac{\partial^{2}C(t,T)}{\partial T^{2}}\right) - \left(\frac{\partial^{2}C(t,T)}{\partial t_{1}\hat{T}}\right)^{2} > 0$$
...(18)

and

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The optimal solution of the equations (16) and (17) can be obtained by using the software Mathematical 5.2. This has been illustrated by the following numerical example.

 $+ \frac{c\mu^{\beta+4}}{\beta+} + \frac{at^{\beta+1}}{\beta\beta+} + \frac{mt^{\beta+2}}{\beta\beta+} + \frac{m\mu^{\beta+2}}{\beta+} - \frac{art^{\beta+2}}{\beta+\beta+} - \frac{mrt^{\beta+2}}{\beta+}$

NUMERICAL EXAMPLE

To illustrate the developed model, an example with the following data has been considered. a = 50 units, s = Rs.25.0 per unit per unit time, b = 20 units, h = Rs.1.0 per unit per unit time, c = 10 units, d = Rs.20.0 per unit per unit time, C = Rs.200 per order, $\ell = Rs.10.0$ per unit per unit time, $\epsilon = 1.2$, $\mu = 1$, $\alpha = 0.10$,

 $\beta = 3.$

Based on these input data, the outputs are:

 $t_1 = 2.10668$, T = 3.03871, Q = 300.367 and $C_A = \text{Rs.}3608.80$.

We studied the effects due to change in parameters α , β , μ and \in on the optimal values of t_1 , T, Q and C_A .

| Effect of scale parameter (α): | | | | | | | | |
|-----------------------------------------|-----------|-----------------------|---------|-----------------------|---------|--|--|--|
| | α | t_1 | Т | Q | C_A | | | |
| | 0.08 | 2.20583 | 3.88188 | 364.301 | 4291.64 | | | |
| | 0.09 | 2.16104 | 3.44555 | 332.919 | 3967.65 | | | |
| | 0.10 | 2.10668 | 3.03871 | 300.367 | 3608.80 | | | |
| | 0.11 | 2.03841 | 2.65399 | 265.392 | 3192.13 | | | |
| | 0.12 | 1.94605 | 2.27687 | 225.665 | 2674.87 | | | |
| | 0.13 | 1.80182 | 1.87763 | 176.6 <mark>54</mark> | 1972.90 | | | |
| | | | | | | | | |
| Effect shape pa | rameter (| [β]: | | | | | | |
| | β | t_1 | Т | Q | C_A | | | |
| | 2.9 | 2.16113 | 3.22139 | 319.922 | 3797.19 | | | |
| | 3.0 | 2.10668 | 3.03871 | 300.367 | 3608.80 | | | |
| | 3.1 | 2.05423 | 2.86633 | 281.740 | 3421.70 | | | |
| | 3.2 | 2.00346 | 2.70348 | 263.944 | 3234.87 | | | |
| | 3.3 | 1.95403 | 2.54952 | 246.895 | 3047.38 | | | |
| | 3.4 | 1.90 <mark>563</mark> | 2.40387 | 230.524 | 2858.47 | | | |
| | | | | | | | | |
| Effect of parameter (μ): | | | | | | | | |
| | μ | | Т | Q | C_A | | | |
| | 0.80 | 2.11103 | 2.99753 | 289.653 | 3431.23 | | | |
| | 0.90 | 2.10891 | 3.01919 | 295.121 | 3520.87 | | | |
| | 1.00 | 2.10668 | 3.03871 | 300.367 | 3608.80 | | | |
| | 1.10 | 2.10437 | 3.05633 | 305.392 | 3695.22 | | | |
| | 1.20 | 2.10201 | 3.07223 | 310.192 | 3780.27 | | | |
| | 1.30 | 2.09964 | 3.08656 | 314.761 | 3864.09 | | | |
| | | | | | | | | |

Effect of backlogging parameter ():

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| E | t_1 | Т | Q | C_A |
|------|---------|---------|---------|---------|
| 1.10 | 1.86481 | 2.37395 | 218.499 | 2083.04 |
| 1.20 | 2.10668 | 3.03871 | 300.367 | 3608.80 |
| 1.30 | 2.20534 | 3.31765 | 335.477 | 4521.17 |
| 1.40 | 2.26035 | 3.47201 | 354.426 | 5132.44 |
| 1.50 | 2.29274 | 3.56177 | 364.635 | 5540.06 |
| 1.60 | 2.31160 | 3.61311 | 369.556 | 5804.35 |

From the above tables, we observe some interesting facts. We notice that T, quantity ordered Q and total average cost C_A of the system decrease with the increment in parameters α and β while these parameters increase with the increment in parameters μ and ϵ . Parameter t_1 decreases with the increment in parameters α , β and μ while t_1 increases with the increment in parameter ϵ

CONCLUSION

This paper deals an order level inventory model for deteriorating items with a new pattern of variable demand which is quadratic initially and becomes linear later on i.e. ramp type demand. The nature of demand of seasonal and fashionable products is increasing-steady-decreasing and becomes asymptotic. The demand pattern assumed here is found to occur not only for all types of seasonal products but also for fashion apparel, computer chips of advanced computers, spare parts, etc. The procedure presented here may be applied to many practical situations. Retailers in supermarket face this type of problem to deal with highly perishable seasonal products. An approximated EOQ model is also provided by considering average demand rate under inflation. In this model deterioration rate at any item is assumed to follow two parameter Weibull distribution function of time. Shortages are allowed and partially backlogged with time dependent backlogging rate. Finally, numerical examples on this model highlighting the results.

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